Abstract—The maintenance model for a repairable system is considered. The system will fail due to an internal component failure, or an external arrival of an accident, whichever occurs first. The consecutive threshold values of an external arrival of an accident are monotone geometric. Whenever the inter-arrival time of two successive small repairable breakdowns is smaller than the specified threshold then the system fails and will be replaced. The optimal policy $N^*$ for minimizing the long run average cost per unit time is determined with a numerical example.

Keywords—External Cause, Internal Cause, Maintenance Model, Poisson Process, Shock Models, Terminal Small Repairable Breakdown

Abbreviation—Geometric Process (GP)

I. INTRODUCTION

Consider an automobile system will fail due to an internal cause (component failure) or due to an external arrival of the terminal small repairable breakdown (accident) whichever occurs first. Suppose the automobile system fails due to an external arrival of an accident, whenever the inter-arrival time of two successive small repairable breakdowns is smaller than the specified threshold then the system fails. Because the GP model is a good model for a system that fails due to its internal failure and the $\delta$-shock model is a reasonable model for a system that fails due to an external failure. A GP, $\delta$-shock maintenance model as a combination of the above two failure models for a repairable system. A lot of research has been done in the theory of the GP, and its application to the maintenance problem. Barlow & Prochan (1975) studied the shock model. That describes, whenever a shock arrives, it will cause a random amount of damage to the system. A shock is a deadly shock, when the accumulated amount of damage to the system by the arrival time of the shock exceeds a threshold, and then the system fails. A number of papers on the maintenance of the system subject to shocks were published by Gottlieb (1982), Shanthikumar & Sumita (1983) and others. In Shanthikumar & Sumita (1983), system failure in shock model approach considered, and a sufficient conditions evaluated under which the system failure time is completely monotone. In Gottlieb (1982), the general case analyzed where the failure rate need not be increasing and replacement can be made at any time. Also optimal replacement policy is found and conditions for optimal policies are determined.

Lam (1988 A), first introduced the Geometric process and its application to maintenance model and in Lam (1988), discussed the monotonicity optimal replacement policy using GP models. In Lam (1991), he discussed the successive operating times of a system decreasing while the consecutive repair times after failure will be increasing for a deteriorating models. Wu & Clements-Croome (2005) developed the maintenance policies for the system subject to shocks were published by Gottlieb (1982), Shanthikumar & Sumita (1983) and others. In Shanthikumar & Sumita (1983), system failure in shock model approach considered, and a sufficient conditions evaluated under which the system failure time is completely monotone. In Gottlieb (1982), the general case analyzed where the failure rate need not be increasing and replacement...
infected case in SARS (Severe Acute Respiratory Syndrome). Tang & Lam (2006) model paid attention to the frequency of the shocks. Yeh Lam (2013), recently studied the monotonicity properties of the optimal replacement policy for an improving system, proved that the optimal replacement policy is the 8 policy, i.e., the policy without replacement.

In practice, the operating time of a system is a random variable because the system can fail due to an external arrival of an accident. In this paper a GP maintenance model is studied. The optimal policy \( N^* \) for minimizing the long run average cost per unit time is determined. When there exists no shock, the successive operating time of the system after repair forms GP. The shocks will arrive according to Poisson process. The frequency of breakdowns rather than the accumulated amount of damage of breakdowns. Whenever the inter arrival time of two successive small repairable breakdowns is smaller than a pre-specified threshold the system fails, and later shock is called a terminal small repairable breakdown. Thus in a \( \delta \) – shock model, a shock is a terminal small repairable breakdown if the time between two breakdowns is less than a specified threshold \( \delta \) (the threshold value), and the system fails at the time of the occurrence of the terminal small repairable breakdown. In this model an external arrival of a terminal small repairable breakdown considered and optimal policy determined to minimize the Long run average cost.

\section{II. Notations}
\begin{itemize}
\item \( \omega \) – Rate of Poisson process of the arrival of shocks
\item \( \lambda \) – Changing rate of a terminal small repairable breakdown
\item \( \delta \) – Threshold of a terminal repair of a new system
\item \( \gamma \) – Rate of Poisson process of the arrival of breakdown
\item \( \alpha \) – Constant ratio of GP for the successive operating time
\item \( \beta \) – Constant ratio of GP for the consecutive repair time
\item \( \zeta \) – The repair cost has a constant rate
\item \( \theta \) – Reward rate, \( R \) – The replacement cost
\item \( a \) – The replacement time, \( \mu_p \) – Constant rate
\item \( P_n \) – The operating time of a system following \((n-1)\)th failure
\item \( Q_n \) – The repair time of the system after \( n \)th failure
\item \( N \) – A policy by which the system will be replaced following the \( N \)th failure.
\item \( N^* \) – Minimizing the long run average cost
\item \( C(N) \) – Long run expected cost, \( g(N) \) – An auxiliary function
\item \( \theta = E(P_{11}), \theta_n = E(P_n), \zeta = E(Q_n), \Psi = E(a) \)
\end{itemize}

\section{III. Assumption of the Model}
\begin{enumerate}
\item At the beginning, a new system is new whenever the system fails it will be repaired. The system will be replaced by a new, identical one following the \( N \)th failure.
\item A system subject to attacks from sequence of repairs. The repair will arrive according to a Poisson process with rate \( \omega \).
\end{enumerate}

\section{IV. Long Run Average Cost}

A cycle is a time interval between the installation of a system and the first replacement, or a time interval between two consecutive replacements. Therefore, the successive cycles will form renewal process. Consequently, the successive cycles together with the cost incurred in each cycle will constitute a renewal processes [Shanthikumar & Sumita, 1983].

The long-run average cost per unit time is calculated using the following formula,

\[\text{Expected cost incurred in a cycle}\]

\[\text{Expected length of a cycle}\]

Moreover, suppose a replacement policy \( N \) is adopted. Let the Long run average cost be denoted by \( C(N) \).

\[C(N) = \frac{1}{\gamma + (a\beta)/\delta} \sum_{n=1}^{N-1} E(P_n) + \sum_{n=1}^{N} E(Q_n) + a\]

\[\theta_n = E(P_n) = 1/(\gamma + (a\beta)/\delta) \exp(-((\gamma + (a\beta)/\delta)\theta \delta)^{-1})\]

Assume \( \gamma = 0 \), that there is no shock, so the system will fail due to the arrival of an external terminal small repairable breakdown. Consequently, (4) yields,
\[ \theta_n = E(P_n) = \frac{\theta}{(\alpha \beta)^{n-1}} \]  

(5)

Thus, the GP \( \delta \) shock model reduces to the GP model. Then,

\[
C(N) = \frac{\sum_{n=1}^{N-1} \frac{1}{y^{n-1}} - r \sum_{n=1}^{N-1} \frac{\theta}{(\alpha \beta)^{n-1}} + R + \mu \rho \psi}{\sum_{n=1}^{N-1} (\alpha \beta)^{n-1} + \xi \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} + \rho \psi} \]  

(6)

To determine an optimal policy \( N^* \) for minimizing the average cost, rewrite (6) as \( C(N) = A(N) - r \), Where,

\[
A(N) = \frac{\sum_{n=1}^{N-1} \frac{\theta}{(\alpha \beta)^{n-1}} + \xi \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} + \rho \psi}{\sum_{n=1}^{N-1} (\alpha \beta)^{n-1} + \xi \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} + \rho \psi} \]  

(7)

Then, minimize \( C(N) \) is equivalent to minimize \( A(N) \), and \( A(N+1) - A(N) \) will always positive. The auxiliary function for minimizing \( A(N) \), i.e., \( A(N+1) > A(N) \text{ if } g(N) > 1 \).

\[
g(N) = \frac{(z + r)(\sum_{n=1}^{N-1} \frac{\theta}{(\alpha \beta)^{n-1}} - \theta N + 1 \sum_{n=1}^{N-1} \frac{1}{y^{n-1}} + \rho \psi)}{(R + (\mu \rho + r)\psi)(\theta N + 1)^{N-1} + \rho \psi} \]  

(8)

V. Numerical Example

Consider the parameter values be \( r = 1.2, \alpha = 1.07, \beta = 1, \gamma = 0.97, \theta = 30, \xi = 20, \psi = 10, \delta = 2, \gamma = 0.07, z = 25, r = 100, \mu = 10 \) and \( R = 3500 \).

<table>
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<th>N</th>
<th>C(N)</th>
<th>g(N)</th>
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<th>C(N)</th>
<th>g(N)</th>
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<td>2.1098</td>
</tr>
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</table>

VI. Conclusion

From the above table, it is clear that \( C(5) = -18.886 \) is the unique minimum average cost. Because \( C(N) \) is decreasing when \( N < 5 \), and increasing when \( N > 5 \). Therefore, an optimal replacement policy is \( N^* = 5 \), and also, it is easy to see that 5 is the first integer so that \( g(N) > 1 \). i.e. \( g(5) = 1.0186 > 1 \). This means that the optimal policy to replace the system at the time following the 5th failure. Thus the GP shock model as a maintenance model for an Automobile system that takes into account the effect of an internal cause, or an external cause on the automobile system. This will effectively determine the proper time to replace the system wherever equipment, machinery parts available.

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References


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