Numerical Study of Heat Transfer on a Fully Buried Pipeline Under Steady-Periodic Thermal Boundary Conditions


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Abstract—The steady-periodic heat transfer on buried pipelines for the transportation of majorly hydrocarbons, and their environment is studied. This is the heat transfer regime for shallow water buried pipelines, in the case of lake Victoria in Kenya, in relation to the temperature changes of the seabed during the year. In this study, first, the initially unsteady two dimensional conduction problem is written in a dimensionless form; then, it is transformed into a steady two dimensional problem and solved numerically by means of finite difference method. Several values of burying depth and of the radius of the pipeline, as well as of the thermal properties of the soil were considered. The numerical results are represented in form of graphs and tables are compared with those obtained by means of an “approximate method” employed in industrial design which is mainly experimental according to Barletta and others in 2007 as in the case of Mediterranean Sea.

Keywords—Approximate Method; Heat Transfer Coefficient; Kenya Pipeline; Lake Victoria; Steady-Periodic Thermal Boundary Conditions.

I. INTRODUCTION

OFFSHORE buried pipelines are many at times used for the transport of hydrocarbons from extraction sites to refinement plants. The issue of heat transfer from buried pipelines has been widely studied [11]. The analytical expression of the steady-state heat transfer coefficient from and on buried pipelines can be found in [11]. It refers to the boundary conditions of a uniform temperature of the sea bed i.e. of the separation surface between the pipeline and the soil [12]. Heat transfer from a buried pipeline is a real heat conduction problem has many applications in the real world. These applications can be small such as under floor heating system like in the case of Ondol system used in Korea which uses heat generated from cooking stoves [13], to more large scale applications such as in the case of Oil and Gas pipelines either under the sea beds or underground such as in the case of Kenya pipelines. Agriculture has considerable applications on heat transfer in pipelines as well [2].

The designs of these pipelines require the knowledge of the overall heat transfer coefficient from the pipe wall to the environment [7]. In fact, a significant decrease of the fluid temperature could cause the formation of hydrates and waxes which might slow down or even stop the fluid flow [1]. Moreover, the knowledge of the bulk temperature of the fluid in any cross section is necessary to determine the value of the viscosity in that section and, thus, to evaluate the viscous pressure drop along the flow direction [4]. As a consequence, the heat transfer from buried pipelines has been widely studied in the literature [5; 15]. An analytical expression of the steady – state heat transfer coefficient from a fully buried pipe to its environment can be found in [16]. It refers to the boundary condition of a uniform temperature of the seabed, i.e. of the separation surface between soil and water. In these conditions, the thermal power exchanged between the pipeline and its environment, per unit length of the duct, can be expressed as

\[ Q = k(T_a - T_e)A_0 \]  

where \( k \) is the thermal conductivity of the soil, \( T_a \) is the temperature of the external surface of the duct, \( T_e \) is the
The temperature of the seabed and $\Lambda_0$ is a dimensionless heat transfer coefficient, given by

$$
\Lambda_0 (\sigma) = \frac{2\pi}{\arccosh(\sigma)} \quad (2)
$$

In equation (2), $\sigma$ is the ratio between the burying depth of the pipe axis $H$, and the external radius of the pipe, $R$.

Equations (1) and (2) provide reliable results in many cases, but cannot be applied, for instance, in the following circumstance. For shallow – water pipelines, the temperature of the seabed is often affected by the season changes and varies accordingly during the year. With reference to Kisumu beaches of the Lake Victoria, the seabed statistically reaches a minimum value of 20°C in cold seasons of June to September and a maximum of value 29°C in the hot seasons of January to March [14]. Clearly, this temperature change of the seabed may have a strong influence on the thermal power exchanged between the pipeline and its environment [1].

At present, an approximate method is employed in industrial design to take into account this effect. The soil is considered as a semi-infinite solid medium whose surface temperature varies in time according to the law

$$
T = T_m + \Delta T \sin(\omega t) \quad (3)
$$

where $T_m$ is the mean annual temperature of the seabed, $\Delta T$ is the amplitude of the temperature oscillation and $\omega$ is the angular frequency which corresponds to the period of one year, namely

$$
\omega = 0.1991.10^6 \text{s}^{-1} \quad (4)
$$

Thus, by assuming that the pipeline does not modify considerably the temperature distribution in the soil, at a depth $H$ has [3].

$$
T_H = T_m - \Delta T \exp \left( \frac{\omega}{\sqrt{2\alpha}} H \right) \sin \left( \frac{\omega}{\sqrt{2\alpha}} (H - \alpha \tau) \right) \quad (5)
$$

where $\alpha$ is the thermal diffusivity of the soil per unit length. In the approximate method, the thermal power is evaluated as

$$
Q = k(T_a - T_H)\Lambda_0 \quad (6)
$$

i.e by replacing $T_e$ with $T_H$ in equation (1).

The approximate method does not seem reliable because it is not based on rigorous mathematical model. To check the reliability of the method, one can apply it to evaluate the heat transfer between a plane isothermal surface buried at depth $H$ and the surrounding soil, when the temperature of the ground surface varies in time according to equation (3) [8]. In fact, for this case, an analytical expression of the soil temperature field is available in the literature [1; 9], while performing the analysis with reference to standard properties of the soil and report as follows. Considering an isothermal horizontal plane, kept at the constant temperature $T_a$, buried at a depth $H$ from the ground surface, whose temperature varies in time according to equation

$$
T = T_m + \Delta T \sin(\omega t) \quad , \text{as shown in the figure below}
$$

![Figure 1: Isothermal Horizontal Plane Buried at a Depth $H$. The Temperature of the Ground Surface in $X = 0$ Varies according to Equation (3)](image)

Thus, the thermal power per unit area which flows from the isothermal surface to the ground can be evaluated as

$$
q = k \frac{\partial T}{\partial X} \bigg|_{X = H} \quad (A5)
$$

But according to the approximate method, the thermal power exchanged per unit area is

$$
q = \frac{k}{H}(T_a - T_H) \quad (A6)
$$

where $T_H$ is given by equation (5).

Equations (A1) to (A6) above are according to [1].

The comparison between the analytical solution and the ‘approximate method’ shows that the latter yields acceptable results only when very small values of $H$ are considered. Therefore a more reliable method to evaluate the steady-periodic heat transfer from buried pipelines to the environment is needed.

The aim of this project therefore is to find a more accurate method to determine the heat transfer between an offshore buried pipeline and its environment in steady-periodic conditions. First, by introducing suitable auxiliary variables, the unsteady two dimensional conduction problem is transformed into a steady two dimensional problem in the new variables. The steady problem is solved numerically using the finite difference method and analysis of the results by means of matlab software.

### II. Objectives of the Study

1. To transform the unsteady two dimensional condition problem into a steady two dimensional problem by introducing suitable auxiliary variables
and solve the resultant problem using the finite difference method by means of matlab software.

ii. To investigate the temperature distribution around the pipe with varying depth in which the pipe is buried in a steady – periodic condition.

iii. To find out a more accurate and reliable method to determine the heat transfer between an offshore buried pipeline and its environment in steady – periodic conditions.

A computer code capable of simulating steady two-dimensional turbulent high Reynolds number convective flows in an enclosure is required in order to realize the objectives of this study.

### III. Mathematical Model

According to Barletta and his colleagues in 2007, the starting point of our model is derived from Fourier’s Law which specifies that the heat transfer is governed by the equation [1]:

\[ \dot{Q} = -k \nabla u \]  

(7A)

with the following boundary conditions:

where \( Q \) is the heat flux vector per unit length, \( k \) is the heat conductivity of the soil, \( u \) is the temperature throughout the region. Assuming that the temperature of the seabed varies in time according to equations (3) and (4). The computational domain and the boundary conditions are sketched in Figure 1. As shown in the figure, the vertical and bottom boundaries of the computational domain are considered as adiabatic. According to [10], the differential equation to be solved is Fourier equation

\[ T = T_a \quad \text{on the pipe surface}; \]

\[ T = T_m + \Delta T \sin(\alpha t) \quad \text{on the seabed}; \]

\[ \vec{n} \cdot \nabla T = 0 \quad \text{on the vertical and bottom boundaries} \]  

(10)

In steady- periodic regime, the temperature field can be expressed as

\[ T(X,Y,t) = T_0(X,Y) + \int \left[ T_1(X,Y) - T_m \right] \sin(\alpha t) + \int \left[ T_2(X,Y) - T_m \right] \cos(\alpha t) \]  

(11)

By substituting equation (11) in equation (7), one obtains

\[ \alpha \nabla^2 T_0 + \nabla^2 T_1 \sin(\alpha t) + \nabla^2 T_2 \cos(\alpha t) = 0 \]  

(12)

Equation (12) implies

\[ \nabla^2 T_0 = 0, \]

\[ \alpha \nabla^2 T_1 = -\omega (T_2 - T_m), \]

\[ \alpha \nabla^2 T_2 = -\omega (T_1 - T_m), \]

(13)

(14)

(15)

#### Values of the Dimensionless Parameters

Non-Dimensionalization of variables is an automatic procedure for numerical studies of fluid flow. It is important to adopt a suitable non-dimensional scheme to express analytical and experimental results in the most efficient form, and also to make the solution bound. The main aim of non-dimensionalization is to reduce the number of parameters particularly when dealing with incompressible fluids. This is because in incompressible fluids, the number of non-dimensionalized parameters is equivalent to the dimensional parameters.

In technical application, the depth \( H \) of the pipeline axis can range from the duct radius \( R \) to about ten times \( R \). Indeed, the following values of \( \sigma \) will be considered: 1.2; 1.5; 2.0; 4.0; 10.0;

The period of the temperature oscillation is one year; thus \( \omega \) is given by equation (4). The thermal properties of the soil vary in the ranges:

\[ k_{\min} = \frac{1}{3} \frac{W}{m K}, \quad k_{\max} = 3 \frac{W}{m K}, \]  

\[ \rho_{\min} = 1.2 \times 10^3 \frac{kg}{m^3}, \quad \rho_{\max} = 2.5 \times 10^3 \frac{kg}{m^3}, \]  

\[ c_{\min} = 1 \times 10^3 \frac{J}{kg K}, \quad c_{\max} = 2 \times 10^3 \frac{J}{kg K}, \]  

\[ \alpha_{\min} = \frac{k_{\min}}{\rho_{\min} c_{\min}} = 2.0 \times 10^{-7} \frac{m^2}{s}, \]  

\[ \rho_{\min} c_{\max} \]

\[ \alpha_{\max} = \frac{k_{\max}}{\rho_{\min} c_{\max}} = 2.5 \times 10^{-6} \frac{m^2}{s}, \]  

(16)

(17)

(18)

(19)

In offshore buried pipelines for the transport of hydrocarbons, the duct diameter varies from six to forty inches, i.e., the radius varies in the range

\[ R_{\min} = 7.6 \text{ cm}, \quad R_{\max} = 0.51 \text{ m} \]  

(20)

In the next section, we are to discuss the numerical method we are to adopt in solving our partial differential equations developed. The merit for the choice of the method are outlined. The temperature boundary conditions that shape our model are to clearly discussed as well.

#### The Temperature Boundary Conditions

The boundary conditions for \( T_0, T_1 \), and \( T_2 \) are:

\[ T_0 = T_a, \quad T_1 = T_2 = T_m, \quad \text{on the pipe surface} \]

\[ T_0 = T_m, \quad T_1 = T_m + \Delta T, \quad T_2 = T_m, \quad \text{on the seabed}; \]

\[ \vec{n} \cdot \nabla T_0 = 0, \quad \vec{n} \cdot \nabla T_1 = 0, \quad \vec{n} \cdot \nabla T_2 = 0 \quad \text{on the vertical and bottom boundaries} \]

From Barletta and his colleagues in 2007, the thermal power exchanged between the pipeline and the environment can be evaluated by means of the expression [1]

\[ k \left[ \int_{\partial L} \vec{n} \cdot \nabla T_0 \, dt + \int_{\partial L} \vec{n} \cdot \nabla T_1 \, dt + \int_{\partial L} \vec{n} \cdot \nabla T_2 \, dt \right] \sin(\alpha t) + \int_{\partial L} \vec{n} \cdot \nabla T \, dt \cos(\alpha t) \]

(21)

(22)

(23)

(24)

where \( \partial L \) is the circular boundary of the duct, \( dl \) is the infinitesimal arc of this boundary and \( \vec{n} \) is the unit vector orthogonal to \( \partial L \), as illustrated in figure 1.

Introducing the following dimensionless variables:

\[ \theta_0 = \frac{T_0 - T_m}{\Delta T}, \quad \theta_1 = \frac{T_1 - T_m}{\Delta T}, \quad \theta_2 = \frac{T_2 - T_m}{\Delta T}, \]

\[ x = \frac{X}{R}, \quad y = \frac{Y}{R}, \quad \vec{v} = R \vec{v}, \]

\[ \Omega = \frac{\alpha \omega^2}{\alpha}, \quad \Xi = \frac{\Delta T}{T_a - T_m} \]

(25)
Equations (13) - (15) then (21) – (23) can be written in the dimensionless form
\[
\nabla^2 \theta_0 = 0; \quad (26)
\]
\[
\nabla^2 \theta_1 = \Omega \theta_2; \quad (27)
\]
\[
\nabla^2 \theta_2 = \Omega \theta_1 \quad (28)
\]
\[
\theta_0 = \frac{1}{\Xi}, \quad \theta_1 = 0, \quad \theta_2 = 0 \text{ on the pipe surface}; \quad (29)
\]
\[
\theta_0 = 0, \quad \theta_1 = 1, \quad \theta_2 = 0 \text{ on the seabed}; \quad (30)
\]
\[
\vec{n} \cdot \nabla \theta_0 = 0, \quad \vec{n} \cdot \nabla \theta_1 = 0, \quad \vec{n} \cdot \nabla \theta_2 = 0, \text{ on the vertical and bottom sides} \quad (31)
\]
by means of equation (25), one can write equation (24) in the form
\[
Q = k(T_a - T_m) \Xi \left[ \int R \nabla \theta_0 d l' + \int R \nabla \theta_1 d l' \sin(\alpha t) + \int R \nabla \theta_2 d l' \cos(\alpha t) \right] \quad (32)
\]
The boundary value problem for the dimensionless temperature field \( \theta_0 \) is the steady conduction problem already solved in [16]. Therefore, the first integral in equation (32) is such that
\[
\Xi \int R \nabla \theta_0 d l' = \Lambda_0 (\sigma) = \frac{2 \pi \arccosh(\sigma)}{\Omega} \quad (33)
\]
In order to determine the dimensionless fields \( \theta_1 \) and \( \theta_2 \), one can solve numerically the coupled differential equations (22) and (23), with the boundary conditions prescribed in equations (24) to (26). The solution depends on the dimensionless parameters \( \sigma \) and \( \Omega \).

By employing the dimensionless heat transfer coefficient \( \Lambda_0 \) and the dimensionless quantities
\[
A = \frac{1}{\Lambda_0} \int R \nabla \theta_1 d l', \quad B = \frac{1}{\Lambda_0} \int R \nabla \theta_2 d l' \quad (34)
\]
One can rewrite equation (32) in the form
\[
Q = k(T_a - T_m) \Lambda_0 \left[ 1 + \Xi \left[ A \sin(\alpha t) + B \cos(\alpha t) \right] \right] \quad (35)
\]
Since \( \Lambda_0 \) is known, equation (35) allows one to evaluate the time – dependent thermal power exchanged between the pipeline and its environment [6].

IV. THE RESULTS AND DISCUSSIONS

4.1. The Temperature Fields
In this chapter, the results and discussions of the temperature distribution are presented in the figures 3 and 4 and tables 1.2 and 3 below. The value of the coefficients \( A \) and \( B \) given by equation
\[
A = \frac{1}{\Lambda_0} \int R \nabla \theta_1 d l', \quad B = \frac{1}{\Lambda_0} \int R \nabla \theta_2 d l' \quad (36)
\]
are determined by matlab software as shown below.

The \( x \) – component and \( y \) – component of the gradient of \( \theta_1 \) and \( \theta_2 \) are multiplied by the \( x \) – components and \( y \) – component of the unit vector normal to the boundary 5 as in figure 2. The latter components are built-in variables denoted as \( nx \) and \( ny \). Then, \( A \) and \( B \) are recorded in tables 1 for \( \sigma = 1.3 \), in table 2 for \( \sigma = 3 \), in table 3 for \( \sigma = 5 \). In order to illustrate the effect \( s \) of the temperature oscillations at the seabed on the heat transfer rate between the pipeline and its environment, let us compare the thermal power exchanged in steady – periodic regime, given by equation (35), with two limiting cases

Table 1: Values of \( A \) and \( B \) for \( \sigma = 1.3 \) and \( \sigma = 1.6 \)

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 1.3 )</td>
<td>( \sigma = 1.6 )</td>
<td></td>
</tr>
<tr>
<td>0.0003</td>
<td>-0.9921</td>
<td>0.00914</td>
</tr>
<tr>
<td>0.001</td>
<td>-0.9855</td>
<td>0.01186</td>
</tr>
<tr>
<td>0.01</td>
<td>-0.9592</td>
<td>0.02889</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.9224</td>
<td>0.04590</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.9010</td>
<td>0.05424</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.8762</td>
<td>0.06358</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.8599</td>
<td>0.06984</td>
</tr>
</tbody>
</table>

4.1.1. Case 1
In this case there is a limit of very high values of \( H \) and very low values of \( \alpha \). From the table 1 above, for very high values of both \( \sigma \) and \( \Omega \), the effect of the temperature oscillations at the seabed on the heat transfer rate becomes negligible. In this limit, the seab temperature oscillations affect only a thin soil layer close to the seabed, far from the pipe, so that the thermal power exchanged between the pipeline and its environment is constant and is given by

\[
Q_{\text{const}} = k(T_a - T_m) \Lambda_0 \quad (36)
\]

Table 2: Values of \( A \) and \( B \) for \( \sigma = 3 \) and \( \sigma = 5 \)

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 3 )</td>
<td>( \sigma = 5 )</td>
<td></td>
</tr>
<tr>
<td>0.0003</td>
<td>-0.9789</td>
<td>0.02329</td>
</tr>
<tr>
<td>0.001</td>
<td>-0.9623</td>
<td>0.02986</td>
</tr>
<tr>
<td>0.01</td>
<td>-0.8979</td>
<td>0.07042</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.8096</td>
<td>0.1112</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.7584</td>
<td>0.1334</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.6968</td>
<td>0.1607</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.6580</td>
<td>0.1803</td>
</tr>
</tbody>
</table>

4.1.2. Case 2
As from table 2, in the limit of \( \sigma \) very close to 1 and of very low values of \( \Omega \), one can note that the temperature oscillations have a uniform phase. The thermal power exchanged between pipeline and environment can be evaluated, at any time instant, considering steady-state conditions with a temperature at the seabed given by the temperature at that instant.
Table 3: Values of $A$ and $B$ for $\sigma = 6$ and $\sigma = 10$

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$A$</th>
<th>$B$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0003</td>
<td>-0.9289</td>
<td>0.07171</td>
<td>-0.8814</td>
<td>0.1106</td>
</tr>
<tr>
<td>0.001</td>
<td>-0.8789</td>
<td>0.8878</td>
<td>-0.8055</td>
<td>0.1345</td>
</tr>
<tr>
<td>0.01</td>
<td>-0.6906</td>
<td>0.1908</td>
<td>-0.5193</td>
<td>-0.2696</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.4435</td>
<td>0.2895</td>
<td>-0.1389</td>
<td>0.3095</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.2833</td>
<td>0.3210</td>
<td>0.03150</td>
<td>0.2136</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.09150</td>
<td>0.2974</td>
<td>0.08847</td>
<td>0.06089</td>
</tr>
<tr>
<td>0.3</td>
<td>0.01494</td>
<td>0.2381</td>
<td>0.06136</td>
<td>-0.00176</td>
</tr>
</tbody>
</table>

Indeed, the results reported in Table 3 above point out that, for $\sigma = 10$ and $\Omega = 0.3$, the values of $A$ and $B$ are very close to zero, confirming that the constant power limit is approached. On the other hand, the results reported in Table 1 above point out that, for $\sigma = 1.2$ and $\Omega = 0.0003$, $A = -1$ and $B \approx 0$, so that the quasi-stationary limit is nearly reached.

The amplitude of the dimensionless temperature oscillations given by $\sqrt{2^{\Omega} + 2^{\Omega_2}}$ is illustrated for $\sigma = 10$ and $\Omega = 0.3$. From the figure, it is clearly shown that the temperature oscillations becomes negligible at the depth of the pipe, so that a nearly constant heat flux from the pipe wall occurs, i.e constant power limit is reached.

The amplitude $\sqrt{2^{\Omega_1} + 2^{\Omega_2}}$ and the phase arctangent $\left(2^{\Omega_2}/2^{\Omega_1}\right)$ of the dimensionless temperature oscillations for $\sigma = 1.3$ and $\Omega = 0.0003$ are illustrated.

The figure shows that important temperature oscillations take place in the soil even at depths greater than that of the pipe, except very close to the pipe boundary, where a constant temperature boundary condition has been imposed.

4.2. Validation of the Results

In order to compare the results presented in this project with the “approximate” results obtainable by equation (6) let us rewrite (35) in the form

$$Q = k(T_a - T_{eff}) \cdot A_0,$$

(37)

Where the effective temperature $T_{eff}$ is given by

$$T_{eff} = T_m - \Delta T \cdot A_0 \left[\left(A \sin(\omega \theta) + B \cos(\omega \theta)\right)\right],$$

(38)

Clearly, equation (6) agrees with equation (37) if $T_m$ coincides with $T_{eff}$.

Equations (5) and (25), one obtains

$$T_m = T_m - \Delta T \exp \left(-\frac{2^{\Omega_1}}{2^{\Omega_2}}\right) \left(-\cos \left(\frac{\Omega}{2^{\Omega_2}}\right) \sin(\omega \theta) + \sin \left(\frac{\Omega}{2^{\Omega_2}}\right) \cos(\omega \theta)\right),$$

(39)

Equations (38) and (39) show that $T_m$ with $T_{eff}$ if

$$A = -\cos \left(\frac{\Omega}{2^{\Omega_2}}\right) \exp \left(-\frac{\Omega}{2^{\Omega_2}}\right),$$

(40)

$$B = \sin \left(\frac{\Omega}{2^{\Omega_2}}\right) \exp \left(-\frac{\Omega}{2^{\Omega_2}}\right),$$

(41)

A comparison between the values $A$ and $B$ obtained numerically in this project and approximate values given by equations (40) and (41) in Figure 4 and Figures 5, where $A$ and $B$ are plotted against the logarithm base 10 of $\Omega$, for $\sigma = 2$, in the range $0.0003 \leq \Omega \leq 0.3$. 

Figure 2: Amplitude for Dimensionless Oscillations for $\sigma = 10$ and $\Omega = 0.3$

Figure 3: The amplitude $\sqrt{2^{\Omega_1} + 2^{\Omega_2}}$ and the phase arctangent $\left(2^{\Omega_2}/2^{\Omega_1}\right)$ of the dimensionless temperature oscillations for $\sigma = 1.3$ and $\Omega = 0.0003$ are illustrated.

Figure 4: A plot of $A$ versus log10 $\Omega$ for $\sigma = 3$ according to equation (34) (blue line) and equation (41) (green line)

Figure 5: A plot of $B$ versus log10 $\Omega$ for $\sigma = 3$ according to equation (34) (blue line) and equation (41) (green line)
The figures 4 and 5 show a strong disagreement between the correct and the “approximate” values of $A$ and $B$ occurs, for high values of $\Omega$. Therefore, the naïve method employed in the industrial design may yield a too rough approximation, except for very low values of $\Omega$.

V. Conclusions

The objective of this study was to find out a more accurate and reliable method to determine the heat transfer between an offshore buried pipeline and its environment in steady–periodic conditions. This was done by transforming the unsteady two dimensional conduction problem into a steady two dimensional problem and solved numerically by means of the finite difference–matlab software.

The results, reported through tables containing the values of two dimensionless parameters, can be employed for a wide range of values of the burying depth and of the radius of the pipeline as well as of the thermal properties of the soil.

The results obtained has been compared with those predicted by an approximate method applied in industrial designs. The comparison revealed that the approximate method employed in industries may be unreliable in some design conditions corresponding to a large diameter of the pipe and to a low thermal diffusivity of the soil.

The project has a wide range of applications in engineering. A good example is in a water heater dirty hot used water flowing through pipes can be used to warm cold water for domestic or industrial use, where direct heating may not be required or is discouraged.

VI. Recommendations

i. There is need to investigate the comparison between the method in this project and the approximate method in other extreme conditions other than the ones discussed in this project.

ii. A more practical approach in engineering would reduce theoretical assumptions made in this work is highly recommended.

References


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Dr. Johanna Kibet Sigey. Sigey holds a Bachelor of Science degree in mathematics (May 2014 to date) besides being a full time teacher of Mathematics and Chemistry at Bishop Mugendi Nyakegogi secondary school (October 2007 to date) in Kisii County, Kenya. He also serves as Deputy director of studies, Head of examinations and Careers and H.O.D (Sciences) in the same institution. He has much interest in the study of heat transfer, fluid flows and MHDS in different mediums and geometries and their respective applications to engineering.
Affiliation: Jomo Kenyatta University of Agriculture and Technology, (JKUAT), Kenya.

He is currently the Director, jkuat, Kisii CBD. He has been the substantive chairman - department of pure and applied mathematics – jkuat (January 2007 to July- 2012). He holds the rank of Associate Professor in Applied Mathematics pure and applied Mathematics department – jkuat since November 2009 to date. He has published 9 papers on heat transfer, MHD and Traffic models in respected journals.

Teaching experience: 2000 to date- postgraduate programmes:
- Master of science in applied mathematics
  - Units: complex analysis I and II, numerical analysis, fluid mechanics, ordinary differential equations, partial differentials equations and Riemannian geometry

Supervision of postgraduate students
- Doctor of philosophy: thesis (3 completed, 5 ongoing)
- Masters of science in applied mathematics: (13 completed, 8 ongoing)

Dr. Okelo Jeconia Abonyo. He holds a PhD in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology as well as a Master of science degree in Mathematics and first class honors in Bachelor of Education, Science; specialized in Mathematics with option in Physics, both from Kenyatta University. He has have dependable background in Applied Mathematics in particular fluid dynamics, analyzing the interaction between velocity field, electric field and magnetic field. Has a hand of experience in implementation of curriculum at secondary and university level. He has demonstrated sound leadership skills and have ability to work on new initiatives as well as facilitating teams to achieve set objectives. He has good analytical, design and problem solving skills.

Affiliation: Jomo Kenyatta University of Agriculture and Technology, (JKUAT), Kenya.

2011-To date Deputy Director, School of Open learning and Distance e Learning SODEL Examination, Admission &Records (JKUAT), Senior lecturer Department of Pure and Applied Mathematics and Assistant Supervisor at Jomo Kenyatta University of Agriculture and Technology. His work involves teaching, research, and assisting in supervision of undergraduate and postgraduate students in the area of applied mathematics. He has published 10 papers on heat transfer in respected journals.

Supervision of postgraduate students
- Doctor of philosophy: thesis (3 completed)
- Masters of science in applied mathematics: (13 completed, 8 ongoing)

Dr. Okwoyo James Mariita. James holds a Bachelor of Education degree in Mathematics and Physics from Moi University, Kenya, Master Science degree in Applied Mathematics from the University of Nairobi and PhD in applied mathematics from Jomo Kenyatta University of Agriculture and Technology, Kenya.

Affiliation: University of Nairobi, Chiromo Campus School of Mathematics P.O. 30197-00100 Nairobi, Kenya. He is currently a lecturer at the University of Nairobi (November 2011 – Present) responsible for carrying out teaching and research duties. He plays a key role in the implementation of University research projects and involved in its publication. He was an assistant lecturer at the University of Nairobi (January 2009 – November 2011). He has published 7 papers on heat transfer in respected journals.

Supervision of postgraduate students
- Masters of science in applied mathematics: (8 completed,8 ongoing)