Numerical Investigation of Flow in a Container in which there is No Geostrophic Contours

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***Abstract*—**The geostrophic mode which exists when the boundary of a container can be covered with geostrophic contours, e.g., the sphere, the ellipsoid, and in the general sense a cylinder has been studied. The study considered numerical investigation of the problem of flow in a container in which there are no geostrophic contours, namely the sliced container. In this study, the equations satisfied by the Rossby waves have been derived and used to find whether there is effect of Rossby number, Cariolis force and Ekman number on flow in a container in which there are no geostrophic contours. The resulting Eartel governing equation has then been solved numerically by Matlab software. From the simulated results it has been found that for a particular time, velocity flow in cylindrical container; increases with increase in Cariolis force, Ekman number and increases with a decrease in Rossby number. It was also found that velocity flow in a sliced cylinder decreases with increases in cylinder height for Cariolis, Rossby number and Ekman number.

***Keywords*—**Cariolis Force and Ekman Number; Eartel's Equation; Geostrophic Contours; Finite Difference Method; Rossby Number; Sliced Container.

***Abbreviations*—**Explicit Difference Scheme (EDS); Partial Differential Equation (PDE).

I. INTRODUCTION AND LITERATURE REVIEW

IN this section, introduction of the area of study is done, literature review, geometry of problem, statement of problem, objectives, basic definitions and significance of study are looked at.

1.1. Introduction

Man has attempted to gain some understanding of the behaviour of the ocean and the atmosphere, with the impetus for such work coming from the need to predict the motion of the water and air that surround us. In ancient times knowledge came almost entirely from records of practical observation, but the last century has seen great advances in the theoretical, numerical and experimental techniques which are used to study this important branch of science. Geophysical fluid dynamics, in its broadest sense, is the study of fluid motions in the earth. The purpose of this study is to give a Mathematical description of a certain class of such phenomena. It is concerned with those problems for which the length scale is sufficiently large that the earth's rotation has a significant effect on the dynamics of the fluid. Hence it excludes many interesting small scale problems, for example, those connected with surface tension, but discuss the Mathematics that describes basic models for the motion of the ocean and the atmosphere. Besides the relevance to geophysics, the research is an appealing one to a Mathematician because the partial differential equations which arise frequently display interesting and rather unusual properties.

Mathematical analysis is developed in an ordered fashion, studying first the equations that describe the simplest Physics, namely small perturbations from the equilibrium of a homogeneous inviscid rotating fluid. It then proceeds from this base to add layer upon layer of Mathematical complexity as further relevant physical factors are included in the model.

The problem of steady flow in a cylinder, which is geotropically free, is degenerate because the height is a constant. In this case the geostrophic equations give less information, and will be satisfied by any pressure. However, it is possible to solve the problem by introducing the Ekman boundary layer. In the case where the boundary is geotropically blocked it is impossible to obtain exact geostrophic balance and the steady geostrophic mode breaks down into an infinite set of low frequency waves known as the Rossby waves.

Geostrophic motion occurs when the horizontal components of the pressure gradient and Cariolis forces are in approximate balance and frictional effects are confined to these boundary layers.

The circulation flows in a sliced container occurs yet there are no geostrophic contours around which a column of fluid can move and retain a constant height, hence the geostrophic mode can no longer exist. In this report, the role of Rossby number, coriolis forces and Ekman number to circulation flows has been studied.

1.2. Geometry of the Problem

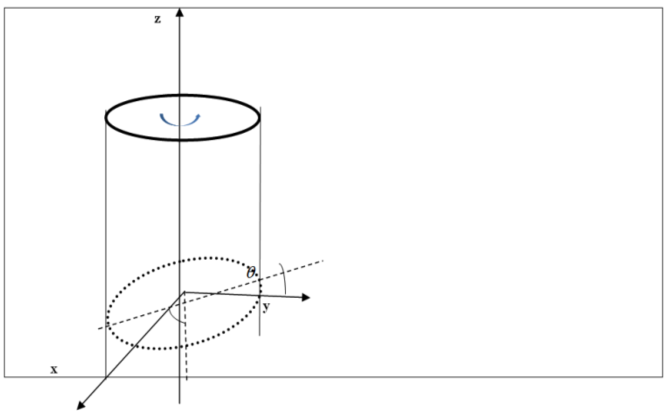

Figure 1: The Flow Geometry

The problem of flow in a container in which there are no geostrophic contours, namely the sliced cylinder has been investigated. In this geometry there are no contours around which a column of fluid can move and retain a constant height; hence the geostrophic mode can no longer exist. However, it has to be shown that the inertial modes carry no mean circulation: thus if the initial flow possesses mean circulation, it must excite a model that is distinct from the inertial modes. It has been shown that in the absence of geostrophic contours, the coriolis force, Ekman number and Rossby number play a significant role in circulation.

1.3. Definition of Terms

1.3.1. Height Contours

These are isolines which denote the distribution of equal height of a particular atmospheric pressure on a constant pressure map.

1.3.2. Geostrophic Wind

This is a steady horizontal motion of air a long a straight, parallel isobars or contours in unchanging pressure or contour field.

1.3.3. Cariolis Force

This is the apparent deflection of moving objects when motion is described relative to a rotating reference frame.

In non-vector terms: at a given rate of rotation of the observer, the magnitude of the Coriolis acceleration of the object is proportional to the velocity of the object and also to the sine of the angle between the direction of movement of the object and the axis of rotation.

The Coriolis effect is the behavior added by the Coriolis acceleration. This implies that the Coriolis acceleration is perpendicular both to the direction of the velocity of the moving mass and to the frame's rotation axis. So in particular:

i) if the velocity is parallel to the rotation axis, the Coriolis acceleration is zero.
ii) if the velocity is straight inward to the axis, the acceleration is in the direction of local rotation.
iii) if the velocity is straight outward from the axis, the acceleration is against the direction of local rotation.
iv) if the velocity is in the direction of local rotation, the acceleration is outward from the axis.
v) if the velocity is against the direction of local rotation, the acceleration is inward to the axis.

1.3.4. Geostrophic Current

This refers to an oceanic flow in which the pressure gradient force is balanced by cariolis force effect.

1.3.5. Causes Geostrophic Current

The Coriolis effect exists only when one uses a rotating reference frame. In the rotating frame it behaves exactly like a real force (that is to say, it causes acceleration and has real effects). However, the Coriolis force is a consequence of inertia_and is not attributable to an identifiable originating body, as is the case for electromagnetic or nuclear forces, for example. From an analytical viewpoint, to use Newton's second law in a rotating system, the Coriolis force is mathematically necessary, but it disappears in a non-accelerating, inertial frame of reference. For example, consider two children on opposite sides of a spinning roundabout (Merry-go-round), who are throwing a ball to each other. From the children's point of view, this ball's path is curved sideways by the Coriolis effect. Suppose the roundabout spins counter-clockwise when viewed from above. From the thrower's perspective, the deflection is to the right from the non-thrower's perspective, deflection is to left.

An observer in a rotating frame, such as an astronaut in a rotating space station, very probably will find the interpretation of everyday life in terms of the Coriolis force accords more simply with intuition and experience than a cerebral reinterpretation of events from an inertial standpoint. For example, nausea due to an experienced push may be more instinctively explained by the Coriolis force than by the law of inertia. In meteorology, a rotating frame (the Earth) with its Coriolis force provides a more natural framework for explanation of air movements than a non-rotating, inertial frame without Coriolis forces. In long-range gunnery, sight corrections for the Earth's rotation are based upon the Coriolis force. These examples are described in more detail below.

The acceleration entering the Coriolis force arises from two sources of change in velocity that result from rotation: the first is the change of the velocity of an object in time. The same velocity (in an inertial frame of reference where the normal laws of physics apply) will be seen as different velocities at different times in a rotating frame of reference. The apparent acceleration is proportional to the angular velocity of the reference frame (the rate at which the coordinate axes change direction), and to the component of velocity of the object in a plane perpendicular to the axis of rotation. .

The second is the change of velocity in space. Different positions in a rotating frame of reference have different velocities (as seen from an inertial frame of reference). In order for an object to move in a straight line, it must therefore be accelerated so that its velocity changes from point to point by the same amount as the velocities of the frame of reference. The effect is proportional to the angular velocity (which determines the relative speed of two different points in the rotating frame of reference), and to the component of the velocity of the object in a plane perpendicular to the axis of rotation (which determines how quickly it moves between those points).

1.3.6. Length Scales and the Rossby Number

The time, space and velocity scales are important in determining the importance of the Coriolis effect. Whether rotation is important in a system can be determined by its Rossby number which is the ratio of the velocity of a system to the product of the Coriolis parameter, and the length scale of the motion:

The Rossby number is the ratio of inertial to Coriolis forces. A small Rossby number signifies a system which is strongly affected by Coriolis forces, and a large Rossby number signifies a system in which inertial forces dominate. For example, in tornadoes, the Rossby number is large, in low-pressure systems it is low and in oceanic systems it is around one. As a result, in tornadoes the Coriolis force is negligible, and balance is between pressure and centrifugal forces. In low-pressure systems, centrifugal force is negligible and balance is between Coriolis and pressure forces. In the oceans, all three forces are comparable.

1.3.7. Application of Coriolis Force to Earth

An important case where the Coriolis force is observed is the rotating Earth. Unless otherwise stated, directions of forces and motion apply to the Northern Hemisphere.

1.4. Intuitive Explanation

As the Earth turns around its axis, everything attached to it turns with it (imperceptibly to our senses). An object that is moving without being dragged along with this rotation travels in a straight motion over the turning Earth. From our rotating perspective on the planet, its direction of motion changes as it moves, bending in the opposite direction to our actual motion. When viewed from a stationary point in space above, any land feature in the Northern Hemisphere turns counter-clockwise, and, fixing our gaze on that location, any other location in that hemisphere will rotate around it the same way. The traced ground path of a freely moving body traveling from one point to another will therefore bend the opposite way, clockwise, which is conventionally labeled as “right,” where it will be if the direction of motion is considered “ahead” and “down” is defined naturally. Cloud formations in a famous image of Earth make similar circulation directly visible.

Perhaps the most important impact of the Coriolis effect is in the large-scale dynamics of the oceans and the atmosphere. In meteorology and oceanography, it is convenient to postulate a rotating frame of reference wherein the Earth is stationary. In accommodation of that provisional postulation, the centifugal and Coriolis forces are introduced. Their relative importance is determined by the applicable Rossby numbers. Tornadoes have high Rossby numbers, so, while tornado-associated centrifugal forces are quite substantial, Coriolis forces associated with tornadoes are for practical purposes negligible.

Because ocean currents are driven by the movement of wind over the water's surface, the Coriolis force also affects the movement of ocean currents and cyclones as well. Many of the ocean's largest currents circulate around warm, high-pressure areas called gyres. Though the circulation is not as significant as that in the air, the deflection caused by the Coriolis effect is what creates the spiraling pattern in these gyres. The spiraling wind pattern helps the hurricane form. The stronger the force from the Coriolis effect, the faster the wind will spin and pick up additional energy, increasing the strength of the hurricane.

Air within high-pressure systems rotates in a direction such that the Coriolis force will be directed radially inwards, and nearly balanced by the outwardly radial pressure gradient. As a result, air travels clockwise around high pressure in the Northern Hemisphere and counter-clockwise in the Southern Hemisphere. Air within low-pressure systems rotates in the opposite direction, so that the Coriolis force is directed radially outward and nearly balances an inwardly radial pressure gradient.

1.5. Flow around a Low-Pressure Area

If a low-pressure area forms in the atmosphere, air will tend to flow in towards it, but will be deflected perpendicular to its velocity by the Coriolis force. A system of equilibrium can then establish itself creating circular movement, or a cyclonic flow. Because the Rossby number is low, the force balance is largely between the pressure gradient force acting towards the low-pressure area and the Coriolis force acting away from the center of the low pressure.

Instead of flowing down the gradient, large scale motions in the atmosphere and ocean tend to occur perpendicular to the pressure gradient. This is known as geostrophic flow. On a non-rotating planet, fluid would flow along the straightest possible line, quickly eliminating pressure gradients. Note that the geostrophic balance is thus very different from the case of "inertial motions" (see below) which explains why mid-latitude cyclones are larger by an order of magnitude than inertial circle flow would be.

This pattern of deflection, and the direction of movement, is called Buys-Ballot Law. In the atmosphere, the pattern of flow is called a cyclone. In the Northern Hemisphere the direction of movement around a low-pressure area is counter-clockwise. In the Southern Hemisphere, the direction of movement is clockwise because the rotational dynamics is a mirror image there. At high altitudes, outward-spreading air rotates in the opposite direction Cyclones rarely form along the equator due to the weak Coriolis effect present in this region.

1.6. Literature Review

Padrino & Joseph [7] investigated numerically simulations of the two-dimensional incompressible unsteady Navier–Stokes equations for streaming flow past a rotating circular cylinder are presented in this study. The numerical solution of the equations of motion was conducted with a commercial computational fluid dynamics package which discretizes the equations applying the control volume method. The numerical set-up was validated by comparing results for a Reynolds number based on the free stream of *Re* = 200 and dimensionless peripheral speed of $\tilde{q}$ = 3, 4 and 5 with results obtained by other authors.

David & Rossen [1] in their study explored several possibilities for modelling weakly nonlinear Rossby waves in fluid of constant depth, which propagate predominantly in one direction. The model equations obtained include the BBM equation, as well as the integrable KdV and Degasperis-Procesi equations.

Rodina & Radu [8] investigated the nonlinear self-adjointness of the nonlinear inviscid barotropic nondivergent vorticity equation in a beta-plane. It is a particular form of Rossby equation which does not possess variational structure and it is studied using a method developed by Ibragimov. The conservation laws associated with the infinite-dimensional symmetry Lie algebra models were constructed and analyzed. Based on this Lie algebra, some classes of similarity invariant solutions with nonconstant linear and nonlinear shears were obtained. It was also shown how one of the conservation laws generates a particular wave solution of this equation.

Omar & Rafat [6] investigated the solitary wave solutions of the (2+1)-dimensional regularized long-wave (2DRLW) equation which is arising in the investigation of the Rossby waves in rotating flows and the drift waves in plasmas and (2+1) dimensional Davey-Stewartson (DS) equation which is governing the dynamics of weakly nonlinear modulation of a lattice wave packet in a multidimensional lattice. By using extended mapping method technique, they showed that the 2DRLW-2DDS equations can be reduced to the elliptic-like equation. Then, the extended mapping method is used to obtain a series of solutions including the single and the combined non degenerative Jacobi elliptic function solutions and their degenerative solutions to the above mentioned class of nonlinear partial differential equations (NLPDEs).

Johnson [4] presented a variational principle which characterizes steady motions, at finite Rossby number, of rotating inviscid homogeneous fluids in which horizontal velocities are independent of depth. This was used to construct nonlinear solutions corresponding to stationary patches of distributed vorticity above topography of finite height in a uniform stream. Numerical results are presented for the specific case of a right circular cylinder and are interpreted using a series expansion, derived by analogy with a deformable self-gravitating body. The results show that below a critical free-stream velocity a trapped circular vortex is present above the cylinder and a smaller patch of more concentrated vorticity, of the opposite sign, maintains a position to the right (looking downstream) of the cylinder. An extension to finite Rossby number and finite obstacle height of Huppert's [1975] criterion for the formation of a Taylor column is presented in an appendix.

Hart [3] used the software MAPLE to study Ekman layer Edge velocities and found out that different velocities give dramatically different suction laws. Kerswell [5] studied inertial oscillations in a container and deduced that non-linear effects can lead to significant large scale effects from the waves. Greenspan & Howard [2] considered the effect of small changes of rotation rate of a container on flow (i.e the small Rossby number limit). The smallness permits a linearization about the original uniform state.

1.7. Statement of the Problem

In a sliced cylinder, there are no contours which a column of fluid can move and retain constant height hence geostrophic mode can no longer exist. Much of the work that has been done on this area in the literature dwells on the Rossby waves in fluid of constant depth .Not much has been done on investigating effect of coriolis forces, Ekman number and Rossby number on circulation flows in the absence of geostrophic contours in a sliced container with varying sliced container height. For this reason our study deals with the effect of these parameters on circulation flows in a sliced container.

1.8. Research Objectives

1.8.1. General Objective

The general objective is to investigate the causes of circulation flow in a sliced cylinder in which there are no geostrophic contours.

1.8.2. Specific Objectives

i) To investigate the role of Rossby number, Coriolis force and Ekman number on velocity flow circulation in a sliced container

ii) To establish the relationship between coriolis force, Rossby number, Ekman number and cylinder height

1.9. Justification of the Research

The outcome of this study has provided more insight to the understanding of the role of coriolis forces, Ekman number and Rossby number on geophysical aspects and circulation flows in the absence of geostrophic contours.

The study of the causes of circulation flows in a sliced cylinder has applications in meteorology and oceanography among others. Meteorology is a branch of science concerned with the processes and phenomena of the atmosphere especially as a means of forecasting the weather.

Oceanography on the other hand is the study of seas and oceans. The major divisions of oceanography include the geological attributes of the ocean floor (plate tectonics) and features, physical oceanography which is concerned with the physical attributes of ocean water such as currents and temperature, chemical oceanography which focuses on the chemistry of the ocean waters, marine biology which is the study of flora and fauna and in meteorology the interaction between the atmosphere and the ocean. The horizontal movement of surface water arising from a balance between the pressure gradient force and the Coriolis force is known as the geostrophic flow.

1.10. Null Hypothesis

There is no relationship between Rossby number, Coriolis force and Ekman number on velocity flow circulation in a sliced cylinder.

In the next section, governing equations for the research has been presented.

II. GOVERNING EQUATIONS

In this research, with the governing equations partial differential equations together with mathematical modelling has been used to investigate the role of Ekman number, Rossby number and coriolis forces in circulation flows. Other Navier-Stokes that governing fluid dynamics under the influence of Earth rotation Equations include energy, momentum and energy equations.

2.1. Derivation of Governing Equations

The Navier-Stokes equations of motion can be written in terms of a uniformly rotating coordinate system;

$$\frac{d\vec{q}}{dt} + \rho\nabla.\vec{q} = 0 \tag{1}$$

$$\rho\left[\frac{d\vec{q}}{dt} + 2\Omega\times\vec{q}\right] = -\nabla P + \rho\nabla(G - \frac{\Omega^2 r^2}{2}) + \mu\nabla^2\vec{q} + \frac{\mu}{3}\nabla\nabla.\vec{q} \tag{2}$$

Where

ρ is the density

P is the pressure

G is the gravitaional potential and

μ is the coefficient of viscosity

Eqns. (1) and (2) describe the motion of a viscous fluid in a coordinate system rotating with uniform velocity $\vec{\Omega}$.

Taking the curl of Eqn. (2) and recalling that $\vec{B} = \nabla\times\vec{q}$ and $\frac{d\vec{B}}{dt} = \frac{\partial}{\partial t} + \vec{q}.\nabla$ we get;

$$\frac{\partial\vec{B}}{\partial t} + \nabla\times(\vec{q}.\nabla\vec{q}) + 2\nabla\times(\vec{\Omega}\times\vec{q}) = -\nabla\times\frac{\nabla P}{\rho} + \mu\nabla\times\frac{\nabla^2\vec{q}}{\rho} \tag{3}$$

Eqn. (3) is the vorticity equation.

Now,

$$\nabla\times(\vec{q}.\nabla\vec{q}) = \nabla\times(\vec{B}\times\vec{q} + \nabla\frac{\vec{q}^2}{2}) \text{ again } \frac{\partial\Omega}{\partial t} = 0$$

Hence equation (3) becomes;

$$\frac{\partial}{\partial t}(\vec{B} + 2\vec{\Omega}) + \nabla\times[(\vec{B} + 2\vec{\Omega})\times\vec{q}] = \frac{\nabla\rho + \nabla P}{\rho^2} + \mu\nabla\times\frac{\nabla^2\vec{q}}{\rho} \tag{4}$$

From equation (1) we have $\nabla.\vec{q} = -\frac{1}{\rho}\frac{d\rho}{dt}$ and hence

$$\frac{\partial}{\partial t}(\frac{\vec{B} + 2\vec{\Omega}}{\rho}) - (\frac{\vec{B} + 2\vec{\Omega}}{\rho}).\nabla\vec{q} = \frac{\nabla\rho + \nabla P}{\rho^3} + \frac{\mu}{\rho}\nabla\times\frac{\nabla^2\vec{q}}{\rho} \tag{5}$$

If the fluid be inviscid *i.e.*, $\mu = 0$, Let λ be any scalar quantity such that $\frac{d\lambda}{dt} = 0$ *i.e.*, λ is conserved as a particle moves in the fluid. Taking the scalar product of $\nabla\lambda$ with equation (5) we get;

$$\nabla\lambda.\frac{d}{dt}\left(\frac{\vec{B} + 2\vec{\Omega}}{\rho}\right) - \nabla\lambda.\left\{\left(\frac{\vec{B} + 2\vec{\Omega}}{\rho}\right).\nabla\vec{q}\right\} = \nabla\lambda.\frac{\nabla\rho\times\nabla P}{\rho^3} \tag{6}$$

Since; $\frac{d\lambda}{dt} = \frac{d\lambda}{dt} + \vec{q}.\nabla\lambda = 0$, then equation (6) yields;

$$\frac{d}{dt}\left\{\nabla\lambda.\left(\frac{\vec{B} + 2\vec{\Omega}}{\rho}\right)\right\} = \nabla\lambda.\frac{\nabla\rho\times\nabla P}{\rho^3} \tag{7}$$

Equation (7) is known as the Eartel's equation.

If L, Ω^{-1} and U denote the Length, Time, and Relative Velocity, we can introduce dimensionless coordinates $\vec{r} = L\vec{r}^*$, $t = \frac{1}{\Omega}t^*$, $\vec{q} = U\vec{q}^*$ and $P = \rho\Omega ULP^*$. In case of constant density Eqns. (1) and (2) reduce to;

$$\nabla.\vec{q} = 0 \tag{8}$$

$$\frac{\partial\vec{q}}{\partial t} + \vec{q}.\nabla\vec{q} + 2\vec{\Omega}\times\vec{q} = -\frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2\vec{q} \tag{9}$$

where P is the pressure incorporating $\left(G - \frac{\Omega^2}{2}\right)$.

Substituting the dimensionless coordinates and with further simplification we get;

$$\frac{\partial\vec{q}^*}{\partial t} + R_o(\vec{q}^*.\nabla^*)\vec{q}^* + 2\hat{k}\times\vec{q}^* = -\nabla P^* + E\nabla^{*2}\vec{q}^* \tag{10}$$

Where;

$\hat{k}$ is the unit vector in the axis of rotation,

$R_o = \frac{U}{\Omega L} = \frac{\text{convective acceleration}}{\text{cariolis acceleration}}$ is the Rossby number

$E = \frac{\upsilon}{\Omega L^2} = \frac{\text{viscous force}}{\text{cariolis force}}$ is the Ekman number

$\upsilon = \frac{\mu}{\rho}$ is the kinematic viscosity

If the flow is steady, we obtain;

$$2\hat{k}\times\vec{q} = -\nabla P \tag{11}$$

and equation (8) remains;

$$\nabla.\vec{q} = 0 \quad (12)$$

Hence the momentum equation has reduced to a balance between Cariolis force and the pressure gradient. This is sometimes known as the geostrophic balance and it is a fundamental characterization of rotating flow.

In the next chapter, the method of solution has been provided together with the boundary conditions.

III. METHODS OF SOLUTION

In this chapter, the Eartel partial differential equations together with boundary conditions have been expressed in finite difference form and then solved using MATLAB software.

3.1. Methods

In this study, numerical scheme has been developed and used finite difference method to solve the model equations. The method obtains a finite system of linear or nonlinear algebraic equations from the PDE by discretizing the given PDE and coming up with the numerical schemes analogues to the equation. The equations have been solved subject to the given boundary conditions. MATLAB software was used to generate solution values in this study.

3.2. Discretization of Model Equation

The finite difference technique basically involves replacing the partial derivatives occurring in the partial differential equation as well as in the boundary and initial conditions by their corresponding finite difference approximations and then solving the resulting linear algebraic system of equations by a direct method or a standard iterative procedure. The numerical values of the dependent variable are obtained at the points of intersection of the parallel lines, called mesh points or nodal point.

3.3. Governing Equation

The Eartel's equation that govern the flow inside a sliced cylinder is discretized in the section that follows.

3.3.1. Eartel's Equation

$$\frac{\partial \vec{q}}{\partial t} + R_o \vec{q}.\nabla \vec{q} + 2\vec{\Omega} \times \vec{q} = -\frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2 \vec{q} \quad (13)$$

If we take Rossby number, Ekman number and Cariolis force as R_o, C and E respectively, equation (13) becomes;

$$\frac{\partial \vec{q}}{\partial t} + R_o\left(\vec{q}.\nabla \vec{q}\right) + C = -\frac{1}{\rho}\nabla P + E\nabla^2 \vec{q} \quad (14)$$

The explicit scheme for the equation above is obtained as follows

$$\frac{\vec{q}_{i,j+1} - \vec{q}_{i,j}}{(\Delta t)} + R_o \frac{\vec{q}_{i+1,j} - \vec{q}_{i-1,j}}{(\Delta x)} + \frac{\vec{q}_{i,j+1} - \vec{q}_{i,j}}{(\Delta y)} + C = \frac{1}{\rho}\frac{p_{i,j} + p_{i-1,j}}{(\Delta x)}$$
$$+E\left(\frac{\vec{q}_{i,j+1} - 2\vec{q}_{i,j} + \vec{q}_{i,j-1}}{(\Delta y)^2} + \frac{\vec{q}_{i,j+1} - 2\vec{q}_{i,j} + \vec{q}_{i,j-1}}{(\Delta x)^2}\right) \quad (15)$$

Taking $C = 30$, $E = 4$, $R_o = 2$ and $\Delta t = \Delta x = 0.05$ arbitrary

3.4. Effect of Rossby Number on Velocity

We investigate the effect of Rossby number on velocity flow in a sliced cylinder. Taking $C = 30$, $E = 4$, $R_o = 2$ and $\Delta t = \Delta x = 0.05$ we get the scheme

$$80\vec{q}_{i+1,j} - 100\vec{q}_{i,j} - 40\vec{q}_{i-1,j} = -3.5 - 60\vec{q}_{i,j+1} + 80\vec{q}_{i,j-1} + P_{i,j} - P_{i-1,j} \quad (16)$$

Taking and i=1, 2, 3…….8 and j=1 we form the following systems of linear algebraic equations

$$\begin{aligned}
&40\vec{q}_{2,1} - 100\vec{q}_{1,1} - 80\vec{q}_{0,1} = -3.5 - 60\vec{q}_{1,2} + 60\vec{q}_{1,0} + P_{1,1} - P_{0,1}\\
&40\vec{q}_{3,1} - 100\vec{q}_{2,1} - 80\vec{q}_{1,1} = -3.5 - 60\vec{q}_{2,2} + 60\vec{q}_{2,0} + P_{2,1} - P_{1,1}\\
&40\vec{q}_{4,1} - 100\vec{q}_{3,1} - 80\vec{q}_{2,1} = -3.5 - 60\vec{q}_{3,2} + 60\vec{q}_{3,0} + P_{3,1} - P_{2,1}\\
&40\vec{q}_{5,1} - 100\vec{q}_{4,1} - 80\vec{q}_{3,1} = -3.5 - 60\vec{q}_{4,2} + 60\vec{q}_{4,0} + P_{4,1} - P_{3,1}\\
&40\vec{q}_{6,1} - 100\vec{q}_{5,1} - 80\vec{q}_{4,1} = -3.5 - 60\vec{q}_{5,2} + 60\vec{q}_{5,0} + P_{5,1} - P_{4,1}\\
&40\vec{q}_{7,1} - 100\vec{q}_{6,1} - 80\vec{q}_{5,1} = -3.5 - 60\vec{q}_{6,2} + 60\vec{q}_{6,0} + P_{6,1} - P_{5,1}\\
&40\vec{q}_{8,1} - 100\vec{q}_{7,1} - 80\vec{q}_{6,1} = -3.5 - 60\vec{q}_{7,2} + 60\vec{q}_{7,0} + P_{7,1} - P_{6,1}\\
&40\vec{q}_{9,1} - 100\vec{q}_{8,1} - 80\vec{q}_{7,1} = -3.5 - 60\vec{q}_{8,2} + 60\vec{q}_{8,0} + P_{8,1} - P_{7,1}
\end{aligned} \quad (17)$$

Taking the initial and boundary conditions $q(x,t) = 0$, $P(0,t) = P(x,t) = 0$, the above algebraic equations (17) can be written in matrix form as

$$\begin{bmatrix} -100 & 80 & 0 & 0 & 0 & 0 & 0 & 0\\ -40 & -100 & 80 & 0 & 0 & 0 & 0 & 0\\ 0 & -40 & -100 & 80 & 0 & 0 & 0 & 0\\ 0 & 0 & -40 & -100 & 80 & 0 & 0 & 0\\ 0 & 0 & 0 & -40 & -100 & 80 & 0 & 0\\ 0 & 0 & 0 & 0 & -40 & -100 & 80 & 0\\ 0 & 0 & 0 & 0 & 0 & -40 & -100 & 80\\ 0 & 0 & 0 & 0 & 0 & 0 & -40 & -100 \end{bmatrix} \begin{bmatrix} \vec{q}_{1,1}\\ \vec{q}_{2,1}\\ \vec{q}_{3,1}\\ \vec{q}_{4,1}\\ \vec{q}_{5,1}\\ \vec{q}_{6,1}\\ \vec{q}_{7,1}\\ \vec{q}_{8,1} \end{bmatrix} = \begin{bmatrix} -3.5\\ -3.5\\ -3.5\\ -3.5\\ -3.5\\ -3.5\\ -3.5\\ -3.5 \end{bmatrix} \quad (18)$$

The results for varying Rossby number i.e for $R_o = 2, 4, 8$ are put in table 1 in section IV.

3.5. Effect of Cariolis Force on Velocity

We investigate the effect of Cariolis force on velocity flow in a sliced cylinder. Taking $C = 30$, $E = 4$, $R_o = 2$ and $\Delta t = \Delta x = 0.05$ we get the scheme.

$$40\vec{q}_{i+1,j} + 60\vec{q}_{i,j} - 80\vec{q}_{i-1,j} = -3.5 - 60\vec{q}_{i,j+1} + 80\vec{q}_{i,j-1} + P_{i,j} - P_{i-1,j} \quad (19)$$

Taking and $i = 1, 2, 3 \ldots\ldots.8$ and $j = 1$ we form the following systems of linear algebraic equations

$$\begin{aligned}
&40\vec{q}_{2,1} + 60\vec{q}_{1,1} - 80\vec{q}_{0,1} = -3.5 - 60\vec{q}_{1,2} + 60\vec{q}_{1,0} + P_{1,1} - P_{0,1}\\
&40\vec{q}_{3,1} + 60\vec{q}_{2,1} - 80\vec{q}_{1,1} = -3.5 - 60\vec{q}_{2,2} + 60\vec{q}_{2,0} + P_{2,1} - P_{1,1}\\
&40\vec{q}_{4,1} + 60\vec{q}_{3,1} - 80\vec{q}_{2,1} = -3.5 - 60\vec{q}_{3,2} + 60\vec{q}_{3,0} + P_{3,1} - P_{2,1}\\
&40\vec{q}_{5,1} + 60\vec{q}_{4,1} - 80\vec{q}_{3,1} = -3.5 - 60\vec{q}_{4,2} + 60\vec{q}_{4,0} + P_{4,1} - P_{3,1}\\
&40\vec{q}_{6,1} + 60\vec{q}_{5,1} - 80\vec{q}_{4,1} = -3.5 - 60\vec{q}_{5,2} + 60\vec{q}_{5,0} + P_{5,1} - P_{4,1}\\
&40\vec{q}_{7,1} + 60\vec{q}_{6,1} - 80\vec{q}_{5,1} = -3.5 - 60\vec{q}_{6,2} + 60\vec{q}_{6,0} + P_{6,1} - P_{5,1}\\
&40\vec{q}_{8,1} + 60\vec{q}_{7,1} - 80\vec{q}_{6,1} = -3.5 - 60\vec{q}_{7,2} + 60\vec{q}_{7,0} + P_{7,1} - P_{6,1}\\
&40\vec{q}_{9,1} + 60\vec{q}_{8,1} - 80\vec{q}_{7,1} = -3.5 - 60\vec{q}_{8,2} + 60\vec{q}_{8,0} + P_{8,1} - P_{7,1}
\end{aligned} \quad (20)$$

Taking the initial and boundary conditions $q(x,t) = 0$, $P(0,t) = P(x,t) = 0$, the above algebraic equations (20) can be written in matrix form as

$$\begin{bmatrix} 60 & 40 & 0 & 0 & 0 & 0 & 0 & 0 \\ -80 & 60 & 40 & 0 & 0 & 0 & 0 & 0 \\ 0 & -80 & 60 & 40 & 0 & 0 & 0 & 0 \\ 0 & 0 & -80 & 60 & 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & -80 & 60 & 40 & 0 & 0 \\ 0 & 0 & 0 & 0 & -80 & 60 & 40 & 0 \\ 0 & 0 & 0 & 0 & 0 & -80 & 60 & 40 \\ 0 & 0 & 0 & 0 & 0 & 0 & -80 & 60 \end{bmatrix} \begin{bmatrix} \vec{q}_{1,1} \\ \vec{q}_{2,1} \\ \vec{q}_{3,1} \\ \vec{q}_{4,1} \\ \vec{q}_{5,1} \\ \vec{q}_{6,1} \\ \vec{q}_{7,1} \\ \vec{q}_{8,1} \end{bmatrix} = \begin{bmatrix} -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \end{bmatrix} \quad (21)$$

The results for varying Cariolis force i.e for $C = 30, 60, 120$ are put in table 2 in chapter four.

3.6. *Effect of Ekman Number on Velocity*

We investigate the effect of Ekman number on velocity flow in a sliced cylinder. Taking $C = 30$, $E = 4$, $R_o = 2$ and $\Delta t = \Delta x = 0.05$ we get the scheme

$$40\vec{q}_{i+1,j} - 100\vec{q}_{i,j} - 80\vec{q}_{i-1,j} = -3.5 - 60\vec{q}_{i,j+1} + 80\vec{q}_{i,j-1} + P_{i,j} - P_{i-1,j} \quad (22)$$

Taking and $i = 1, 2, 3 \dots \dots .8$ and $j = 1$ we form the following systems of linear algebraic equations

$$\begin{aligned} 40\vec{q}_{2,1} - 100\vec{q}_{1,1} - 80\vec{q}_{0,1} &= -3.5 - 60\vec{q}_{1,2} + 60\vec{q}_{1,0} + P_{1,1} - P_{0,1} \\ 40\vec{q}_{3,1} - 100\vec{q}_{2,1} - 80\vec{q}_{1,1} &= -3.5 - 60\vec{q}_{2,2} + 60\vec{q}_{2,0} + P_{2,1} - P_{1,1} \\ 40\vec{q}_{4,1} - 100\vec{q}_{3,1} - 80\vec{q}_{2,1} &= -3.5 - 60\vec{q}_{3,2} + 60\vec{q}_{3,0} + P_{3,1} - P_{2,1} \\ 40\vec{q}_{5,1} - 100\vec{q}_{4,1} - 80\vec{q}_{3,1} &= -3.5 - 60\vec{q}_{4,2} + 60\vec{q}_{4,0} + P_{4,1} - P_{3,1} \\ 40\vec{q}_{6,1} - 100\vec{q}_{5,1} - 80\vec{q}_{4,1} &= -3.5 - 60\vec{q}_{5,2} + 60\vec{q}_{5,0} + P_{5,1} - P_{4,1} \\ 40\vec{q}_{7,1} - 100\vec{q}_{6,1} - 80\vec{q}_{5,1} &= -3.5 - 60\vec{q}_{6,2} + 60\vec{q}_{6,0} + P_{6,1} - P_{5,1} \\ 40\vec{q}_{8,1} - 100\vec{q}_{7,1} - 80\vec{q}_{6,1} &= -3.5 - 60\vec{q}_{7,2} + 60\vec{q}_{7,0} + P_{7,1} - P_{6,1} \\ 40\vec{q}_{9,1} - 100\vec{q}_{8,1} - 80\vec{q}_{7,1} &= -3.5 - 60\vec{q}_{8,2} + 60\vec{q}_{8,0} + P_{8,1} - P_{7,1} \end{aligned} \quad (23)$$

Taking the initial and boundary conditions $q(x,t) = 0$, $P(0,t) = P(x,t) = 0$, the above algebraic equations (23) can be written in matrix form as

$$\begin{bmatrix} -100 & 40 & 0 & 0 & 0 & 0 & 0 & 0 \\ -80 & -100 & 40 & 0 & 0 & 0 & 0 & 0 \\ 0 & -80 & -100 & 40 & 0 & 0 & 0 & 0 \\ 0 & 0 & -80 & -100 & 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & -80 & -100 & 40 & 0 & 0 \\ 0 & 0 & 0 & 0 & -80 & -100 & 40 & 0 \\ 0 & 0 & 0 & 0 & 0 & -80 & -100 & 40 \\ 0 & 0 & 0 & 0 & 0 & 0 & -80 & -100 \end{bmatrix} \begin{bmatrix} \vec{q}_{1,1} \\ \vec{q}_{2,1} \\ \vec{q}_{3,1} \\ \vec{q}_{4,1} \\ \vec{q}_{5,1} \\ \vec{q}_{6,1} \\ \vec{q}_{7,1} \\ \vec{q}_{8,1} \end{bmatrix} = \begin{bmatrix} -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \\ -3.5 \end{bmatrix} \quad (24)$$

The results for varying Ekman number i.e for $E = 4, 8, 16$ are put in table 3 in section four.

In the next section IV, a detailed discussion of solution results of equation governing the velocity flow in a sliced cylinder is presented.

IV. Research Results and Discussion

In this section, matrices equations (18), (21) and (23), together with boundary conditions expressed in finite difference form and then solved using MATLAB software in section III results are presented in tables and graphs. The results on how the Rossby number, Ekman number and Cariolis force affect the velocity profiles in a sliced cylinder have been presented in tabular and graphical form.

4.1. *Results for Effect of Rossby Number on Velocity Profiles*

We solve equation (18) using matlab and get the results as in table 1 below

Table 1: Velocity Profiles $\vec{q}(x,t)$ values for varying Rossby Number R_o

	R_o=2	R_o=4	R_o=8
X=0	7.10126 ×10^{-2}	13.824×10^{-2}	14.34023×10^{-2}
X=1	6.40152×10^{-2}	21.5482×10^{-2}	25.53579×10^{-2}
X=2	5.6275×10^{-2}	27.98913×10^{-2}	33.21348×10^{-2}
X=3	4.76146×10^{-2}	30.98913×10^{-2}	37.11739×10^{-2}
X=4	4.088937×10^{-2}	30.98913×10^{-2}	37.11739×10^{-2}
X=6	3.416245×10^{-2}	27.98913×10^{-2}	33.21348×10^{-2}
X=7	2.664774×10^{-2}	21.5482×10^{-2}	25.53579×10^{-2}
X=8	1.81409×10^{-2}	13.824×10^{-2}	14.34023×10^{-2}

The results in the table 1 above is represented graphically as seen in figure 2 below.

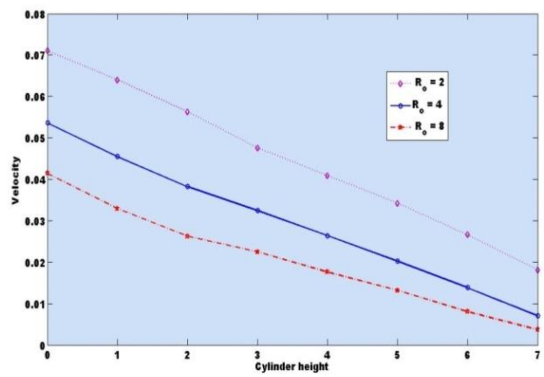

Figure 2: Graph of Velocity against Cylinder Height at Varying Rossby Number

The effect of Rossby number can be observed from figure 1. It is seen that an increase in Rossby number leads to decreases in velocity flow in the cylinder. As the height of the cylinder increases, the velocity flow decreases. A small Rossby number signifies a system which is strongly affected by Coriolis forces, and a large Rossby number signifies a system in which inertial forces dominate.

4.2. *Effect of Coriolis Force on Velocity Profiles*

We solve equation (21) using matlab and get the results as in table 2 below.

Table 2: Velocity Profiles $\vec{q}(x,t)$ Values for Varying Cariolis Force C

	C=30	C=60N	C=120N
X=0	7.50126 ×10^{-2}	13.824×10^{-2}	14.34023×10^{-2}
X=1	5.00152×10^{-2}	21.5482×10^{-2}	25.53579×10^{-2}
X=2	5.6275×10^{-2}	27.98913×10^{-2}	33.21348×10^{-2}
X=3	5.16146×10^{-2}	30.98913×10^{-2}	37.11739×10^{-2}
X=4	4.888937×10^{-2}	30.98913×10^{-2}	37.11739×10^{-2}
X=6	4.316245×10^{-2}	27.98913×10^{-2}	33.21348×10^{-2}
X=7	3.464774×10^{-2}	21.5482×10^{-2}	25.53579×10^{-2}
X=8	2.11409×10^{-2}	13.824×10^{-2}	14.34023×10^{-2}

The results in the table 2 above is represented graphically as seen in figure 3 below.

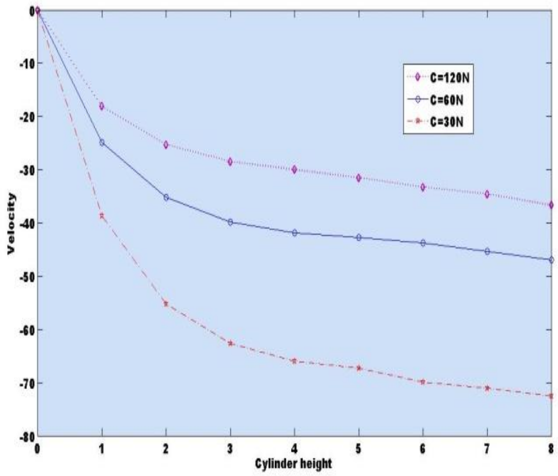

Figure 3: Graph of Velocity against Cylinder Height at Varying Cariolis Force

The effect of Cariolis force can be observed from figure 3. At smaller height of the cylinder, an increase in Cariolis force leads to increases in velocity flow in the cylinder. As the height of the cylinder increases, the velocity flow tend to be constant due to effect of gravitational force decreasing it. This is due to the fact that the magnitude of the Coriolis acceleration of the object is proportional to the velocity of the object and also to the angle between the direction of movement of the object and the axis of rotation.

4.3. *Effect of Ekman Number E on Velocity Profiles*

We solve equation (24) using matlab and get the results as in table 3 below.

Table 3: Velocity Profiles $\vec{q}(x,t)$ Values for Varying Ekman Number E

	E=4	E=8	E=16
X=0	7.50126×10^{-2}	13.824×10^{-2}	14.34023×10^{-2}
X=1	5.00152×10^{-2}	21.5482×10^{-2}	25.53579×10^{-2}
X=2	5.6275×10^{-2}	27.98913×10^{-2}	33.21348×10^{-2}
X=3	5.16146×10^{-2}	30.98913×10^{-2}	37.11739×10^{-2}
X=4	4.888937×10^{-2}	30.98913×10^{-2}	37.11739×10^{-2}
X=6	4.316245×10^{-2}	27.98913×10^{-2}	33.21348×10^{-2}
X=7	3.464774×10^{-2}	21.5482×10^{-2}	25.53579×10^{-2}
X=8	2.11409×10^{-2}	13.824×10^{-2}	14.34023×10^{-2}

The results in the table 1 above is represented graphically as seen in figure 4 below.

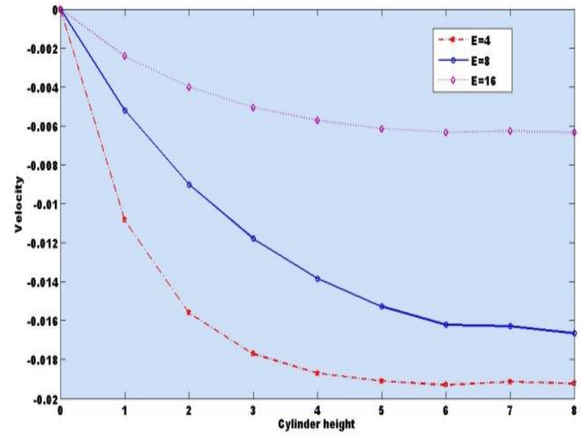

Figure 4: Graph of Velocity against Cylinder Height at Varying Ekman Number

The effect of Ekman number can be observed from figure 4. Increase in Ekman number leads to increases in velocity flow in the cylinder. As the height of the cylinder increases, the velocity flow decreases.

A small Ekman number signifies a system which is strongly affected by Coriolis forces, and a large Ekman number signifies a system in which viscous forces dominate

The next section concludes the research work and gives specific recommendations to further improve on the results.

V. Conclusion and Recommendations

In this section, conclusion and recommendations are made for the study.

5.1. *Conclusion*

Eartel equation was used to quantitatively model the flow in a container in which there is no geostrophic contours. The governing equation and boundary conditions that described the simplified model enabled investigation of the flow in a container problem using Finite Difference Method. From the simulated results we have found that for a particular time,

i) Velocity flow in a sliced cylinder increases with increase in Cariolis and Ekman number but increases with a decrease in Rossby number.
ii) Velocity flow in a sliced cylinder decreases with increases in cylinder height for Cariolis, Rossby number and Ekman number

5.2. *Recommendations*

Further work is recommended to improve on the results so far obtained for velocity flow in a sliced cylinder. This may be done by;

i) Investigate flow in a sliced cylinder in which there is geostrophic contours.
ii) Investigate the effect of density of fluid in a sliced cylinder for velocity flow in which there is no geostrophic contours.

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